Chapter 6 - Structural Analysis of Alternatives

1 Development of Alternatives

Alternatives are investment possibilities. The initial step is to identify which alternatives exist, then to define the alternatives in terms of timing and amounts of present and future cash flows, as well as determining a minimum acceptable rate of return (MARR) and specifying future objectives, resources and constraints.

2 Classification of Alternatives

2.1 Independent Alternatives

Acceptance of one alternative in the set has no effect on acceptance of any other alternative in the set. Accepted if proposal meets the criteria of acceptability, as long as capital is available.
Example. Consider the following independent projects:

Proposal	**Initial Cost**	**Life**	**Annual Cash Flow**	**NPW (at $i = 5\%$)**
N	\$1000	5	\$300	+\$299
E	\$2000	5	\$400	−\$268
W	\$3000	5	\$900	+\$896
S	\$4000	5	\$1000	+\$329

If the criterion is a rate of return $\geq 5\%$, proposals N, S, and W would be satisfactory.

2.2 Dependent Alternatives

Alternatives are related in some way. These can be divided into two types:

- – Mutually Exclusive - choice of one alternative implies other alternatives cannot be selected. For these decisions, net present worth, future worth, net annual worth, or incremental IRR could be used.
  – Contingency-Dependent - choice of one alternative requires the choice of another alternative, as they are linked through physical, legal, political, or administrative requirement. For these decisions, it is best to combine all dependent alternatives into a single alternative.

Example. Suppose proposal W in the above example can only be implemented if E was also implemented. Then we should combine W and E into a new single alternative E+W:

Proposal	Initial Cost	Life	Annual Cash Flow	NPW (at $i = 5\%$)
E+W	$2000 + 3000 = \$5000$	5	$400 + 900 = \$1300$	+\$628

If E is not contingent on the choice of W, and N and S are independent alternatives, the alternative set would be

$$N, E, E + W, S$$

3 IRR Analysis of Mutually Exclusive Alternatives

Decision process for evaluating mutually exclusive alternatives:

- Step 1: Calculate the IRR for the lowest cost alternative (if only relative costs are available, assume the lowest cost alternative is acceptable and skip to step 3).
- Step 2: Compare the IRR with the MARR to determine if alternative is acceptable. If it is not, find the next highest IRR and compare again to the IRR, until an acceptable alternative is found and then continue to step 3.
- Step 3: Determine the next increment of investment.
- Step 4: Compare the IRR of the increment of investment to the MARR. If $IRR < MARR$, eliminate the alternative; otherwise it is acceptable.
- Step 5: If there is no higher level of investment, go to step 6. Otherwise, calculate the IRR for the increment between the last acceptable alternative and the next higher level of investment and go to step 4.
- Step 6: Choose that alternative (if capital is available).

Example. Four designs for a product yield expected costs and revenues as follows:

	Design A	Design B	Design C	Design D
Initial Investment	170,000	260,000	300,000	330,000
Annual Revenues	114,000	120,000	130,000	147,000
Annual Costs	70,000	71,000	64,000	79,000

If a minimum rate of return of 10% is required and a 10-year study period is used, which of the 4 designs is the most attractive?

The alternatives have already been ordered in terms of investment requirements. The incremental values we will need to solve this problem are

	$A \to B$	$A \to C$	$C \to D$
Additional investment	$260,000 - 170,000 = 90,000$	130,000	30,000
Annual net return	$49,000 - 44,000 = 5,000$	22,000	2,000

Step 1. For design A, the IRR is such that

$$-170,000 + (114,000 - 70,000)(P/A, i, 10) = 0$$

Interpolating from the tables in Appendix D, the IRR for this investment is 22.5%.
Step 2. The IRR for design A exceeds the MARR of 10%, so design A is acceptable.
Step 3. The increment of investment from A to B is $\$260,000 - 170,000 = \$90,000$, and net annual returns are \$5,000.
Step 4. The incremental IRR of moving from A to B is clearly negative, since 10 years of \$5,000 returns would not cover the \$90,000 incremental investment even at $i = 0\%$. The incremental IRR is such that

$$-90,000 + 5,000(P/A, i, 10) = 0$$

which yields $i = -10\%$,which is less the MARR so design B is not acceptable.
Step 5. Evaluate design C relative to design A. Incremental IRR of moving from A to C is found by

$$-130,000 + 22,000(P/A, i, 10) = 0$$

which yields a IRR value of 10.9% by interpolation and Appendix D tables.
Step 4. Since the incremental IRR of C exceeds the MARR, C becomes the acceptable alternative.
Step 5. The incremental IRR of moving from C to D is found by

$$-30,000 + 2,000(P/A, i, 10)$$

Again, this IRR is negative and so design D is not acceptable. Since there are no alternatives remaining, we continue to step 6.
Step 6. Choose design C (assuming \$300,000 is available for investment).

If there is limited capital, step 5 can be modified to read "If there is no higher level of investment or there is insufficient capital for the next higher investment, go to step 6."

The IRRs on each investment for the above problem are as follows:

	IRR
Design A	22.5%
Design B	13.5%
Design C	17.7%
Design D	15.9%

Choosing the alternative with the highest rate of return on its investment (design A) would have been a mistake, as the additional investment for design C would yield 10.92%, a gain of almost one percent over the 10% gained on other investments (selecting A would result in a loss of 0.92% on the \$130,000 compared to what could have been earned with design C).

A second common error would be to select the largest investment with an IRR larger than the MARR (design D). This would mean the extra \$30,000 investment in design D instead of C would earn *less* than the 10% it could have earned elsewhere.

4 Compatibility of Incremental IRR Analysis with Other Measures

To achieve the same conclusions as in incremental IRR analysis:

- in the above steps, we could have replaced IRR with present worth (PW), future worth (FW) or equivalent annual worth (EAW) - instead of $IRR > MARR$, the criteria for acceptability of increments would have been $PW > 0$, $FW > 0$ or $EAW > 0$.
- we could have also evaluated alternatives on *total investment* PW, FW or EAW instead of increments.

For example, the following table lists the PW, FW and EAW of total investments for the 4 designs above.

Design	PW	FW	EAW
A	100,361	260,310	16,334
B	41,084	106,561	6,686
C	105,541	273,746	17,177
D	87,831	227,811	14,295

5 Analysis of Independent Alternatives

5.1 Capital Budgeting Limitations

Suppose we have 4 independent proposals (1,2,3,4) and a total limit on funding of \$2300, $i = 10\%$

Proposal	Cost	Year 1 Revenue	Year 2 Revenue	Year 3 Revenue	IRR	PW
1	300	130	130	130	14.4%	23.29
2	500	210	210	210	12.5%	22.24
3	600	250	250	250	12.0%	21.71
4	1500	628	628	628	12.3%	61.74

If we rank the alternatives by IRR,

Proposal	IRR	Proposal Cost	Cumulative Costs
1	14.4	300	300
2	12.5	500	800
4	12.3	1500	2300
3	12.0	600	2900

If \$2300 is available, we should fund proposals 1, 2 and 4. Ranking by PW would have given the same result in this case but this is not generally the case. In reality, the algorithm to select among proposals would be more complex than the ranking by IRR.

5.2 Ranking by PW to Initial Cost Ratio

Another possibility is ranking on the basis of the ranking ratio

$$\frac{PW(MARR)}{Cost}$$

Using the MARR of 10% and the above 4 proposals, this ranking would be

Proposal	**Ranking Ratio**
1	$\frac{23.29}{300} = 0.078$
2	$\frac{22.24}{500} = 0.045$
4	$\frac{61.74}{1500} = 0.041$
3	$\frac{21.71}{600} = 0.036$

This ranking is by chance the same as by the IRR method.

5.3 Mutually Exclusive Groups of Independent Alternatives

Proposals which satisfy the MARR can be grouped into every potential combination of projects. Each group would then be mutually exclusive. We can indicate proposals in a particular group by 1 and those not in the group by 0.

For the above 4 proposals we have 15 possible groups (plus one null group), with the following total costs and annual revenues:

Group	1	2	3	4	Group Cost	Group Annual Revenues
0	0	0	0	0	0	0
1	1	0	0	0	300	130
2	0	1	0	0	500	210
3	1	1	0	0	800	340
4	0	0	1	0	600	250
5	1	0	1	0	900	380
6	0	1	1	0	1100	460
7	1	1	1	0	1400	590
8	0	0	0	1	1500	628
9	1	0	0	1	1800	758
10	0	1	0	1	2000	838
11	1	1	0	1	2300	968
12	0	0	1	1	2100	878
13	1	0	1	1	2400	1008
14	0	1	1	1	2600	1088
15	1	1	1	1	2900	1218

Since proposals 13, 14 and 15 exceed our budget of \$2300, these can be eliminated. To evaluate groups 0 to 12, we can use any of the methods available for evaluating mutually exclusive alternatives. If we use PW, group 11 has the highest group present worth

$$PW_{11} = 23.29 + 22.24 + 61.74 = 107.27$$

as it contains the three highest PW proposals. For many proposals, this method is very cumbersome.

6 Reinvestment-Dependent Alternatives

- Investments are reinvestment-dependent when one investment allows other investment opportunities in the future that would not have been available otherwise.
- PW and incremental IRR methods are sufficient when future investments are independent from current investments
- Analysis of reinvestment-dependent alternatives is achieved by
    1. estimating the rate of return of future cash flows from reinvestments and additional capital investments
    2. determining the amount, timing and future worth of the dependent cash flows, and
    3. discounting the future worth to the present at the MARR.

Example. Consider 3 mutually exclusive alternatives, A (no special reinvestment possibilities), B (annual receipts can be reinvested at $IRR = 20\%$) and C (investment possibility at year 2 of twice original investment amount to earn 30% return). MARR is 10%. The cash flow data for the 3 proposals is

	End of Year	**A**	**B**	**C**
	0	-1000	-1000	-1000
	1	400	600	1100
	2	400	600	-2000
	3	400	0	0
	4	400	0	0
Reinvestment Rate		MARR	20%	30%
PW at 10% (ignoring reinvestments)		268	41	0
FW (reinvested receipts)		0	1901	4844
PW at 10% (including reinvestments)		268	298	656

For proposal B, the two \$600 receipts are reinvested at 20%, for a future worth of reinvestments after 4 years of

$$FW = 600(F/P, 20, 3) + 600(F/P, 20, 2) = \$1901$$

The present value of this project with reinvestments is then

$$PW(B) = -1000 + 1901(P/F, 10, 4) = \$298$$

For proposal C, the \$1100 capital recovery payment is assumed to return only the MARR when reinvested, but the new \$200 investment earns 30% over the remaining 2 years, so that the FW of reinvestments is

$$FW = 1100(F/P, 10, 3) + 2000(F/P, 30, 2) = \$4844$$

The present value of this project is then

$$PW(B) = -1000 - 2000(P/F, 10, 2) + 4844(P/F, 10, 4) = \$656$$

Reinvestment opportunities thus changes the PW ranking of these 3 alternatives (from A-B-C to C-B-A).

7 Supplementary Considerations

- Taxation
- Inflation
- Sensitivity analysis
- Risk analysis