Chapter 5 - Rate of Return Comparisons

1 Rate of Return Methods

1. MINIMUM ACCEPTABLE RATE OF RETURN (MARR) - the lowest level of return that makes an investment acceptable.
2. INTERNAL RATE OF RETURN (IRR) - the rate that makes PW=0 by assuming that all cash flows are re-invested at the IRR.
3. HISTORICAL EXTERNAL RATE OF RETURN (HERR) - the rate that yields a FW=0 by assuming that all positive cash flows are re-invested at the MARR.
4. EXPLICIT REINVESTMENT RATE (ERR) - designated interest percentage appropriate for a specific application.

2 MARR - Minimum Acceptable Rate of Return

MARR is the lowest limit for investment acceptability. It is a device designed to make the best use of money.

There is no agreement on what should be the MARR. Its lower bound should be the cost of capital. This differs across organizations. How much MARR exceeds the cost of capital depends on the circumstances facing an organization. The effect of setting a MARR is to ration capital among different uses.

3 IRR - Internal Rate of Return

The best known and most widely used rate of return method. The IRR can be calculated by equating the PW (or FW) of cash flows to zero and solving for the interest rate. Solving in this way may result in a polynomial equation with multiple roots. In such cases, the IRR may or may not be one of the roots.

3.1 Calculation of IRR

The calculation of IRR depends on the type of investment (simple or non-simple) and the characteristics of the alternatives (independent or mutually exclusive).

Independent Alternatives: the choice of one does not affect the choice of another, except for limited capital availability. Here, each alternative is evaluated based on its own merit using any of the choice criteria such as PW, FW, EAW, EAC, IRR.

Mutually Exclusive Alternatives: the selection of one precludes the selection of others. Here, if the IRR is used, it has to be the "**incremental IRR**" analysis in order to be consistent with the PW, FW or AW rankings.

Simple Investment: there is only one cash flow change from period to period in the cash flow sequence. For simple investments, there is only one root to the equation for Net PW and this root is the IRR. Thus, we find i^* such that

$$Net\ PW(i^*) = PW\ (\mathrm{Re}\,ceipts, i^*) - PW(Disbursements, i^*) = 0$$

Non-Simple Investment: more than one sign change in the cash flow sequence. There may be multiple roots (i^* values) in this case. To find the true IRR we may use several methods. One is the ERR; another is the HERR. Yet another is the Project Balance Method.

3.2 Single, Simple Investment

3.2.1 Income Producing Proposal

A piece of land can be purchased now for $80,000 and is expected to be worth $150,000 within 5 years. During this period, it can be rented at $1500 per year. Annual taxes are $850 per year. What rate of return will be earned on this investment if the estimates are accurate?

Find i^*so that

$$150000(P/F, i^*, 5) + 1500(P/A, i^*, 5) = 80000 + 850(P/A, i^*, 5)$$

which reduces to

$$150000(P/F, i^*, 5) - 80000 + 650(P/A, i^*, 5) = 0$$

To find i^*, we proceed by trial and error. At $i = 0\%$,

$$150000(P/F, 0, 5) - 80000 + 650(P/A, 0, 5) = \$73250$$

and at $i = 15\%$,

$$150000(P/F, 15, 5) - 80000 + 650(P/A, 15, 5) = -\$3244.14$$

Therefore, i^* lies somewhere between 0% and 15%. Try $i = 14\%$:

$$150000(P/F, 14, 5) - 80000 + 650(P/A, 14, 5) = \$136.95$$

and so i^* lies between 14% and 15%. We can then use linear *interpolation:*

Range of i = 15%-14% = 1%
Range of PW=$136.95-(-3244.14)

$$i^* = 14\% + 1\%\frac{136.95 - 0}{3381.09} = 14.04\%$$

Note: if PW>0, employ higher i; conversely, lowering i increases PW.

3.2.2 Cost Reduction Proposal

Subassemblies for a model IV scope cost \$71 each. The annual demand is 350, and it is expected to continue for 3 years (at which time the model V will be ready). With equipment purchased and installed for \$21000, the production cost to internally produce the subassemblies should be \$18500 for the first year and \$12500 for the remaining 2 years. The equipment has no salvage value. Should the company buy or make the subassemblies?

$$\begin{aligned}&\text{Present Annual Cost} = 350(71) = \$24850\\ &\text{Net Savings, Year 1} = \$24850 - 18500 = \$6350\\ &\text{Net Savings, Years 2, 3} = \$24850 - 12250 = \$12,600\end{aligned}$$

$$PW = -21000 + 6,350(P/F, i, 7) + 12,600(P/F, i, 2) + 12,600(P/F, i, 3)$$

We need to find i^* such that PW=0.

At $i = 10\%$	PW=\$4652.62
At $i = 15\%$	PW=\$2,333.89
At $i = 20\%$	PW=\$333.78
At $i = 25\%$	PW=-1404.80

Therefore, $20\% < i^* < 25\%$. By interpolation:

$$i^* = IRR = 20\% + 5\% \frac{333.78 - 0}{333.78 - (-1404.80)} = 21\%$$

If the MARR is less than 21%, manufacture the subassemblies internally; otherwise buy them.

3.3 Consistency of IRR with AW and PW

Suppose a function is currently performed at a labour cost of \$20000.

PLAN A	Do nothing
PLAN B	Invest \$30000 to reduce labour costs to \$15000 for the next 10 years
PLAN C	Install a labour-saving device for \$25000 that will cut labour costs to \$12000. The device will last 5 years, with no salvage value.

If the company's MARR is 8%, which plan offers the greatest benefit?

$$EAC(A) = \$20000$$

$$\begin{aligned} EAC(B) &= \$30000(A/P, 8, 10) + 15000 \\ &= \$30000(0.14903) + 15000 = \$19471 \end{aligned}$$

$$\begin{aligned} EAC(C) &= 25000(A/P, 8, 5) + 12000 \\ &= 25000(0.25046) + 12000 = \$18,262 \end{aligned}$$

Plan C, with the lowest annual cost, is preferred. The \$25000 investment will yield 8% per year plus the additional receipt of \$20000-18262=\$1738 per year savings in labour costs for 5 years. To find the PW of *costs* in each plan, we use $(P/A, 8, 10) = 6.71008$ in $PW = EAC(P/A, 8, 10)$

$$PW(A) = 20000(6.71008) = \$134202$$
$$PW(B) = 19471(6.71008) = \$130652$$
$$PW(C) = 20000(6.71008) = \$122539$$

The PW criterion also chooses Plan C. Given that money is worth 8%, Plan C will cost \$134202-122539=\$11663 less than Plan A over 10 years. This is the PW of the annual gain

$$1738(P/A, 8, 10) = \$1728(6.71008) = \$11662$$

3.3.1 IRR Comparison

The three plans above are mutually exclusive, so we use incremental IRR analysis. Positive cash flow stream is developed from the savings generated by each additional increment of investment.

Incremental IRR: assumes that we start with the smallest acceptable investment. Analysis of a higher-investment alternative is then based on the difference between the cash flows of the two alternatives. If the *incremental* cash flow is acceptable when compared to the MARR, then the larger investment is better. Otherwise, we remove the larger alternative from consideration. This continues until all alternatives have been evaluated. One of the mutually exclusive alternatives is then determined to be the best investment.

Compare Plans A (no investment) and C (\$25000 investment):

By adopting C, labour costs fall by \$8000 (\$20000-12000). Find the incremental IRR i^* such that

$$(0 - \$25000) + 8000(P/A, i^*, 5) = 0$$

Solving this equation yields

$$(P/A, i^*, 5) = \frac{25000}{8000} = 3.125$$

Interpolating between $i = 15\%$ and $i = 20\%$

$$i^* = IRR = 15\% + 5\% \frac{3.352 - 3.125}{3.352 - 2.991} = 18.03\%$$

Since this is greater than the MARR of 8%, Plan C is chosen over Plan A.

Next, compare Plan C with Plan B:

By adopting B instead of C, labour costs *increase* by \$3000 per year. Find i^* such that

$$(25000 - 30000) - 3000(P/A, i^*, 5) = 0$$

The incremental PW of adopting plan B over C will be negative regardless of the interest rate. This implies that the incremental IRR for plan B over C is less than the MARR and C should be selected.

3.3.2 IRR Misconceptions

IRR analysis should be done correctly, otherwise misconceptions can easily arise.

Example: Consider the following cash flow profiles for mutually exclusive projects X and Y with 4 year lives and no salvage values ($MARR = 10\%$):

	0	1	2	3	4
Project X	-1000	100	350	600	850
Project Y	-1000	1000	200	200	200

Using the PW criterion:

$$\begin{aligned} PW(X) &= -1000 + [100 + 250(A/G, 10, 4)](P/A, 10, 4) \\ &= -1000 + [100 + 250(1.38117)](3.16987) = \$411.52 \end{aligned}$$

$$\begin{aligned} PW(Y) &= -1000 + [1000 + 200(P/A, 10, 3)](P/F, 10, 1) \\ &= -1000 + [1000 + 20092.48685)](0.90909) = \$361.24 \end{aligned}$$

So the PW method ranks project X higher than project Y.

Using individual IRRs:

For project X, find i^*_Xsuch that

$$PW(X) = -1000 + [100 + 250(A/G, i^*_X, 4)](P/A, i^*_X, 4) = 0$$

For $i^*_X = 20\%$, $PW(X) = 83.53$ and for $i^*_X = 25\%$, $PW(X) = -40.64$. By interpolation,

$$i^*_X = 20\% + 5\% \frac{83.53 - 0}{83.53 - (-40.64)} = 23.4\%$$

For project Y, find i^*_Y such that

$$PW(Y) = -1000 + [1000 + 200(P/A, i^*_Y, 3)](P/F, i^*_Y, 1)$$

By trial and error and interpolation, $i^*_Y = 34.26\%$. Individual IRR ranks Y higher than X. However, since the projects are mutually exclusive, we must use incremental analysis to remove the inconsistency. Incremental IRR of project X over Y:

Find the rate of return such that $PW(i^*) = PW(X, i^*) - PW(Y, i^*) = 0$

$$PW(i^*) = -900 + 150(P/F, i^*, 1) + 400(P/F, i^*, 2) + 650(P/F, i^*, 3) = 0$$

Solution of this by trial and error gives and incremental IRR of 12.8%, which is greater than the MARR so X is chosen. However, if $12.8\% < MARR < 34.26\%$, project Y would have been chosen, and if $MARR > 34.26\%$, neither project would be chosen. So, for relatively low interest rates, the larger future cash flows of project X make it more attractive than Y, but as the interest rate increase, these future flows become less and less significant (smaller present values) and Y can be preferred.

3.4 Non-Simple Investment

More than one possible rate of return.

Example: One of the alternatives for improving an operation is to do nothing for 2 years and then spend \$10000 for improvements. The gain from this action is \$3000 followed by 2 years of break-even operations, and thereafter annual income of \$2000 per year for 4 years. What rate of return can be expected from this action?

$$PW = 3000 - 10000(P/F, i, 2) + 2000(P/A, i, 4)(P/F, i, 2)$$

At $i = 0\%$, $PW = 3000 - 10000 + 2000(4) = \$1000$
At $i = 10\%$, $PW = 3000 - 10000(0.82645) + 2000(3.1698)(0.82645) = -\$25$
At $i = 51\%$, $PW = 3000 - 10000(0.43906) + 2000(1.5856)(0.43906) = \$2$

Here there are two sign reversals in PW, confirming there are 2 roots to the above polynomial equation. By trial and error we find the 2 roots are roughly $i^* = 9.4\%$ and $i^* = 51\%$. To resolve the multiplicity problem, we can use the External Reinvestment Rate (ERR) or the Historical Rate of Return (HERR).

4 ERR - Explicit Reinvestment Rate

Here we apply an explicit interest rate to a limited portion of the cash flow in order to eliminate one of the sign reversals. The explicit re-investment rate could be the MARR or a rate suggested by the PW profile.

For the above example, let us assume that the \$3000 receipt at the beginning of year 1 is invested at the explicit rate $i\%$ for the following 2 years. Then the PW equation is modified to

$$PW = 3000(F/P, i\%, 2) - 10000 + 2000(P/A, i^*, 4) = 0$$

For each value of $i\%$ we can find i^*.This yields

Explicit Reinvestment Rate, %	IRR on net investment
0	5.6
5	7.5
9.4	9.5
15	12.3
20	15.6
30	22.6
40	33.1
51	51.2
60	77.6

When funds can be invested at, say, 15%, the $IRR = 12.3\% < 15\%$,and so the investment is not worthwhile. However, if funds can be invested externally at $i\% < 9.4$ or $i\% > 51\%$ the IRR exceeds the external rate and the project is attractive.

5 HERR - Historical External Rate of Return

Here we assume that receipts are re-invested at a generally available interest rate, the HERR. The HERR could typically be taken to be the MARR. Let HERR be denoted by $h\%$. Then we find an unknown rate of return e' such that the FW of receipts compounded at $h\%$ is equal to the FW of disbursements compounded at e' :

$$FW(receipts, h\%) = FW(discursements, e')$$

When $h\% = MARR$ and $e' > h\%$, then the investment is assumed attractive (because it promises a yield greater than the lower limit of acceptability).

For the above example, assume that $MARR = h\% = 15\%$. Then we get

$$3000(F/P, 15, 6) + 2000(F/A, 15, 4) = 10000(F/P, e', 4)$$

so that

$$(F/P, e', 4) = 1.69259$$

By interpolation this gives the value $e' = 14.1\%$. Since this is less than the MARR, the investment is unacceptable under the specific HERR assumptions.