Chapter 4 - Equivalent Annual-Worth Comparisons

Compare projects in terms of annual worth instead of present worth or rate-of-return methods. All three give the same preference among investment alternatives.

1 Capital Recovery Annuity

Annual-worth involves the capital recovery factor which converts a lump sum into an equivalent annuity. The factor $(A/P, i, N)$ accounts for both investment capital P and the interest earned on the uncovered portion of the investment.

Example: Asset purchased for \$40000 with expected life of 4 years. The buyer intends to recover the \$40000 over the 4 years plus the interest that the money would have earned elsewhere. If $i = 10\%$, what is the series of equal payments that would return the capital plus interest?

Equivalent annual payment:

$$\begin{aligned} A &= P(A/P, i, N) = P(A/P, 10, 4) \\ &= 40000(0.31547) = \$12,618.83 \end{aligned}$$

You can go on to trace the recovery of the capital in this problem:

Period	Recovery Charge	Interest on Unrecovered Capital	Recovered	Unrecovered
0				\$40000
1	\$12,618.83	$40000(0.10) = 4000$	\$8618.83	\$31381.17
2	\$12,618.83	$31381.17(0.10) = 3138.12$	\$9480.72	\$21900.45
3	\$12,618.83	$21900.45(0.10) = 2190.05$	\$10428.79	\$11471.66
4	\$12,618.83	$11471.67(0.10) = 1147.17$	\$11471.66	\$0

Unrecovered capital can be determined directly from the present worth of remaining payments. For example, the unrecovered capital at the end of year 3 would be

$$A(P/A, 10, 1) = 12,618.83(0.90909) = \$11471.65$$

or the unrecovered capital at the end of year 2 would be

$$A(P/A, 10, 2) = 12,618.83(1.76554) = \$21900.48$$

If there is a salvage value (S) for the asset, Equivalent Annual Cost (EAC) is found by

$$EAC = P(A/P, i, N) - S(A/F, i, N)$$

or alternatively,

$$EAC = (P - S)(A/P, i, N) + iS$$

The former is more intuitive, but the latter is easier to manipulate.

Example: Asset cost is \$60000 with $S = \$20000$, $N = 4$, $i = 10\%$.

$$\begin{aligned} EAC &= 60000(A/P, 10, 4) - S(A/F, 10, 4) \\ &= 60000(0.31547) - 20000(0.21547) \\ &= 18928 - 4309 = \$14619 \end{aligned}$$

or

$$\begin{aligned} EAC &= (60000 - 20000)(A/P, 10, 4) + (0.10)20000 \\ &= 40000(0.31547) + 2000 \\ &= 12619 + 2000 = \$14619 \end{aligned}$$

Notice that the value of \$12619 is the same as in the no salvage value case, but EAC also includes the \$2000 charged for capital locked in S.

2 Equivalent Annual Worth Comparisons

2.1 Consolidation of Cash Flows

Example: A firm proposes to provide a training program for its employees. The program lasts one year, costs \$2000 per month, and promises potential savings in the first month of \$800 and increases of \$400 per month for the remainder of the year. During the program, costs will increase by \$1200 in the first month and then decline at a rate of \$100 per month. If $i = 12\%$ compounded monthly, and the program must pay for itself within a year, should the firm undertake the training program?

$$i = \frac{r}{m} = \frac{12\%}{12} = 1\%$$

Equivalent monthly worth of savings:

$$800 + 400(A/G, 1, 12) = 800 + 400(5.3682) = \$2947$$

Equivalent monthly worth of costs

$$-2000 + [-1200 + 100(A/G, 1, 12)] = -3200 + 100(5.36815) = -\$2663$$

Therefore, the equivalent net monthly cash flow is

$$\$2947 - 2663 = \$284$$

2.2 Recovery of Invested Capital

Example: Purchase of truck will reduce labour cost by \$10000 per year. Purchase price is \$57000, operating costs are \$100 per month and the salvage value will be \$6000 in 12 years. If $I = 12\%$, should the truck be purchased?

$$EAW = -57000(A/P, 12, 12) + 6000(A/F, 12, 12) = -\$153$$

The purchase of the truck will cause a loss of \$153 per year for the next 12 years, compared with other investments with a 12% return.

2.3 Net Cash Flow Comparison

Sales of a machine will increase by \$23000 if a mobile demo-unit is built. A large unit will cost \$71000, and a small unit would cost \$53000. Salvage values after 5 years would be \$8000 and \$3500, respectively. The large unit will save \$3000 annually but transportation cost will exceed those of the small unit by \$1300. If $i = 15\%$, should a mobile unit be built, and if so, which size?

EAW of Large Mobile Unit:

Annual Increase in Profit	\$23000
Cost Saving	3000
Transportation Cost	-1300
Capital Recovery Cost	-19994
Net EAW	\$4706

As the capital recovery cost of the large mobile unit is

$$(71000 - 8000)(A/P, 15, 5) + 8000(0.15) = \$19994$$

The EAW of the small mobile unit is

Annual Increase in Profit	\$23000
Capital Recovery Cost	-15888
Net EAW	\$7112

As the capital recovery cost of the small mobile unit is

$$(55000 - 3500)(A/P, 15, 5) + 3500(0.15) = \$15888$$

Therefore, we should build the small unit, which has a larger EAW.

3 Asset Life

Asset life:

-Service life (time an asset is kept in service by the owner)

-Accounting life (life expectancy based on bookkeeping and tax considerations)

-Economic life (time period that minimizes the asset's total EAC or maximizes equivalent net annual income)

3.1 Assets with Equal Lives

A machine is needed for 3 years. Purchase price is \$77662. Salvage value is \$25000. A comparable machine can be leased for \$30000 per year. If $i = 20\%$, should we lease or buy?

$$EAC(BUY) = (77662 - 25000)(A/P, 20, 3) + 25000(0.20) = \$30000$$

$$EAC(LEASE) = \$30000$$

If the salvage value of \$25000 is accurate, buying earns 20% on the capital invested and so is preferred to leasing.

3.2 Assets with Unequal Lives

Two machines can be purchased to do a job. The Type-I machine has an initial cost of \$3300, operating costs of \$900 per year, and a 4 year life. The Type-II machine has an initial cost of \$9100, operating costs of \$400 per year and a 8 year life. Both machines have no salvage value at the end of their life. If $i = 8\%$, which machine is preferred?

Type-I capital recovery cost:

$$P(A/P, 8, 4) = 3300(0.30192) = \$996$$

Type-II capital recovery cost:

$$P(A/P, 8, 8) = 9100(0.17402) = \$1584$$

Therefore

$$EAC_I = 900 + 996 = \$1896$$

$$EAC_{II} = 400 + 1584 = \$1984$$

Type-I is preferred since $EAC_I < EAC_{II}$.
Note: If you replace the type-I machine once to extend its life to 8 years, the same conclusion applies.