Chapter 3 - Present Worth Comparisons

Present Worth (PW) is the equivalent present value of a cash flow. PW is useful in making comparisons and choosing among investments.

1 Conditions for PW Comparisons

1. Cash Flows are known with certainty.
2. Cash flows are in constant-value money (no inflation).
3. The interest rate is known with certainty.
4. No taxes.
5. Comparisons do not include intangibles.
6. Comparisons ignore the financing aspects of different projects.

2 Present-Worth Equivalence

Find the PW of different projects and choose the one with the highest PW.

Example: An investor makes 3 annual end-of-year payments of \$15000 starting today, which will generate receipts of \$10000 at the end of year 4 and will increase by \$2500 for the following 4 years. If the investor can earn a rate of return of 10% on an alternative 8-year investment, is this alternative attractive?

Here, receipts and expenses each represent an annuity - one positive and one negative. $i = 10\%$.

$$\begin{aligned} PW &= -15000(P/A, 10, 3) + [10000 + 2500(A/G, 10, 5)](P/A, 10, 5)(P/F, 10, 3) \\ &= -15000(2.4868) + [10000 + 2500(1.81)](3.7907)(0.751) \\ &= -37302 + 41368 = \$4066 \end{aligned}$$

3 Net Present Worth (or Net Present Value)

Choose projects with the highest NPW (or NPV).

$$NPW = PW(Benefits) - PW(Costs)$$

Projects with negative NPW are not worth considering.

Example: Two devices perform a function for 3 years with the following cost and benefit profile:

Year	0	1	2	3
Device A	-9000	4500	4500	4500
Device B	-14500	6000	6000	8000

The interest rate is 8%. Which device should be chosen?

$$\begin{aligned} NPW(A) &= -9000 + 4500(P/A, 8, 3) \\ &= -9000 + 4500(2.577) = \$2597 \end{aligned}$$

$$\begin{aligned} NPW(B) &= -14500 + 6000(P/A, 8, 2) + 8000(P/F, 8, 3) \\ &= -14500 + 6000(1.7832) + 8000(0.79383) = \$2550 \end{aligned}$$

Device A should be chosen, other things being equal.

4 Assets with Unequal Lives

4.1 Common Multiple Method

Alternatives are co-determined by choosing a period that spans a common multiple of the lives of the involved assets. For example, if Asset A lasts 3 years and Asset B lasts 4 years, a common multiple is 12. Asset A is replicated 4 times and Asset B is replicated 3 time. This method assumes replicated assets have similar cost characterisitics.

4.2 Study Period Method

Choose a time period of duration that corresponds to the length of a project or the time period the assets are expected to be in service. The study period will be the length of

(a) the shortest of all competing alternatives
(b) the known duration of required services
(c) the time before a better replacement becomes available

Example: Two assets: A1 and A2. A1 has an initial cost of \$2300, a 3 year life, operating costs of \$250 per year and no salvage value. A2 has an initial cost of \$3200, a 4 year life, no operating costs and a \$400 salvage value. If $i = 15\%$, which asset is preferred by (i) the least common multiple method, and (ii) a 2-year study period?

(i) The common multiple is 12 for this problem.

$$\begin{aligned} PW(A1) &= -2300 - 2300(P/F, 15, 3) - 2300(P/F, 15, 6) - 2300(P/F, 15, 9) \\ &\quad -250(P/A, 15, 12) \\ &= -\$6816 \end{aligned}$$

$$\begin{aligned} PW(A2) &= -3200 - 3200(P/F, 15, 4) - 3200(P/F, 15, 8) + 400(P/F, 15, 12) \\ &= -\$5642 \end{aligned}$$

Asset A2 has the cost advantage of \$6816-5642=\$1174.

(ii) Assuming a salvage value of zero for both assets after 2 years,

$$PW(A1) = 2300 - 250(P/A, 15, 2) = -\$2707$$

$$PW(A2) = \$3200$$

Therefore A1 has the lowest present cost for the 3-year servies period.

What salvage value S would make $PW(A1) = PW(A2)$?

$$\begin{aligned} 2707 &= 3200 - S(P/F, 15, 2) \\ S &= \frac{3200 - 2707}{(P/F, 15, 2)} = \frac{493}{0.75614} = \$652 \end{aligned}$$

Thus, A2 is preferred to A1 if the resale value of A2 at the end of 2 years is more than \$652 greater than the resale value of A1.

5 Assets with Infinite Lives

Capitalized cost is first cost plus the present worth of disbursements assumed to last forever.

CAPITALIZED WORTH $= P + \frac{A}{i}$

where P is first cost and A is the difference between annual receipts and disbursements. This is because

$$\lim_{N\to\infty}(P/A, i, N) = \lim_{N\to\infty} \frac{1}{i}\frac{(1+i)^N - 1}{(1+i)^N} = \frac{1}{i}$$

CAPITALIZED WORTH $= P + \frac{Disbursements}{i}$

6 Deferred Investments

Leasing versus expanding capacity over time.

Example:

Lease a building for 10 years at \$23000 per year, OR

Build a small warehouse now for \$110,000 and make a \$50000 addition to it in 3 years. Annual costs would be \$1000 for each of the first 3 years and \$2000 for each of the remaining 7 years. In 10 years the warehouse will have a resale value of \$50000. If $i = 12\%$, which alternative is preferable?

$$PW(lease) = -23000(P/A, 12, 10) = -\$129955$$

$$\begin{aligned} PW(building) &= -110000 - 50000(P/F, 12, 3) + 50000(P/F, 12, 10) \\ &\quad -1000(P/A, 12, 3) - 2000(P/A, 12, 7)(P/F, 12, 3) \\ &= -\$138389 \end{aligned}$$

Thus the PW of storage costs for the next 10 years will be \$8434 less from leasing than building.

7 Future Worth

In some problems, it is useful to look at future worth (FW) than at present worth (PW).

$$FW = PW(F/P, i, N) = PW(1+i)^N$$

If PW is not known, FW can be computed directly. See section 3.6 of the text.

8 Valuation

VALUE: measure of worth in terms of money or goods.

BOOK VALUE: worth of an asset for accounting purposes (based on historical cost)

MARKET VALUE: the price at which an asset can be currently sold (dependent on earnings)

9 Bond Valuation

Bonds are solds by organizations to raise money. Bonds pay interest and are typically redeemable for a specified value at any given time.

FACE VALUE: amount borrowed and promised to be paid back at maturity

BOND RATE: interest rate paid by the bond. Payments are in the form of an annuity

$$A = \text{premium (payment)} = (\text{FACE VALUE})(\text{BOND RATE})$$

Example: A 10-year bond with a face value of \$1000 has a bond rate of 8% paid *semi-annually*. If a buyer wishes to earn a nominal rate of 12% on the investment, what is the purchase price the buyer is willing to pay?

$$A = 1000\left(\frac{0.08}{2}\right) = \$40$$

The buyer desires 6% per period ($\frac{12\%}{2}$) for 20 periods (10 years), so

$$\begin{aligned} PW &= 1000(P/F, 6, 20) + 40(P/A, 6, 20) \\ &= 1000(0.31181) + 40(11.469) = \$770.57 \end{aligned}$$

In general, the value of a bond depends on the size and timing of interest payments and the time until maturity. Short-term bonds tend to have lower interest rates than long-term bonds: investment in short-term securities expose investors to less chance of fluctuations in market value.

10 Stock Valuation

PREFERRED STOCK: No voting rights, pays fixed dividend per period. The PW of preferred stock is

$$PW = \frac{Dividend}{i}$$

COMMON STOCK: Equity share in a company. More difficult to value than preferred stocks because dividends and prices are not constant over time. It is therefore necessary to forecast future earnings, dividends and the prices of common stock. If reliable forecasts could be made, stock valuation would result from discounting the forecast cash flow.