Chapter 13 - Risk Analysis

1 Overview

Outcomes of economic decisions are uncertain due to uncontrollable factors (chance events).

Risk (the chance of loss) is derived from exposure to chance events, and is quantified on the probabilities of occurance of different cash flows. This leads to decision making under risk.

Uncertainty is when probabilities of outcomes are unknown or meaningless.

2 Recognizing Risk

1. Cash flows are uncertain.
2. Salvage values are uncertain.
3. Interest rates are uncertain.
4. Environmental factors are uncertain.

3 Including Risk in Economic Analyses

(a) Definition of the problem
(b) Collection of the data
(c) Formulating the model
(d) Evaluation
(e) Apporaches to incorporating risk: statistics-based models, simulation

4 Probability Concepts

- objective vs. subjective

4.1 Random Variables

- variables with a probability distribution associated with its values
Example:

i%	**Pr**	**R**	**Pr**
12	0.10	5000	0.05
10	0.70	8000	0.85
7	0.20	10000	0.10

4.2 Distribution Function (Cumulative Probability)

$$F(x) = P(X \leq x)$$

Example:

i%	**Pr(I)**	**F(i)**
7	0.20	0.20
10	0.70	0.90
12	0.10	1.00

4.3 Dependent and Independent Random Variables

For dependent random variables, the value of one variable is influenced by the outcomes of some other variable. For independent random variables, the value of one random variable is not affected by the values taken by any other variable.

4.4 Disjoint Events

The occurance of one event excludes the occurance of the other. For disjoint events, the probabilities are additive:

$$P(A + B + C) = P(A) + P(B) + P(C)$$

If A and B are not disjoint,

$$P(A + B) = P(A) + P(B) - P(A \cap B)$$

For independent events,

$$P(A \cap B) = P(A)P(B)$$

the multiplication rule, where $P(A \cap B)$ is the joint probability.

4.5 Expected Value (average value)

$$E(x) = \sum_i P(x_i) x_i$$

4.6 Variance (dispersion)

$$\begin{aligned} V(x) &= \sum_i P(x_i) \left[x_i - E(x)\right]^2 \\ &= \sum_i P(x_i) x_i^2 - [E(x)]^2 \end{aligned}$$

or

$$V(x) = E(x^2) - [E(x)]^2$$

4.7 Standard Deviation

$$\sigma_x = \sqrt{V(x)}$$

Example:

	P(low)=0.2	P(average)=0.6	P(high)=0.2	EV
Proposal A	900	1000	1100	1000
Proposal B	400	1000	1600	1000

$$V(A) = 0.2(900)^2 + 0.6(1000)^2 + 0.2(1100)^2 - (1000)^2 = 4000$$

$$V(B) = 0.2(400)^2 + 0.6(1000)^2 + 0.2(1600)^2 - (1000)^2 = 144000$$

4.8 Standard Score

$$Z = \frac{X - \mu}{\sigma}$$

If X is normal, Z is normal (0,1)

4.9 Coefficient of Variation

$$CV = \frac{\sigma}{\mu}$$

5 Applications of Probability Concepts

Alternative	Rejection P=0.1	Average P=0.6	Domination P=0.3
A	-50,000	200,000	500,000
B	-200,000	100,000	1,000,000

$$E(A) = 0.1(-50000) + 0.6(200000) + 0.3(500000) = \$265,000$$

$$E(B) = 0.1(-200000) + 0.6(100000) + 0.3(1000000) = \$340,000$$

Therefore, product B is preferred.

6 Expected Value of Risky Alternatives

A process line will continue to be needed for 3 years at an annual cost of $310,000. A new redesign for the line has been suggested. The new approach will cost $150,000 and will cut annual costs to $210,000 for the first year. There is a 50% chance annual costs will remain constant, but there is a 25% probability that annual costs will increase from $210,000 by either $20,000 or $75,000 per year. If the MARR is 12%, should the new design be installed?

$$EAC(P = 0.5) = 150000(A/P, 12, 3) + 210000 = \$272,453$$

$$EAC(P = 0.25) = 150000(A/P, 12, 3) + [210000 + 20000(A/G, 12, 3)] = \$290,945$$

$$EAC(P = 0.25) = 150000(A/P, 12, 3) + [210000 + 75000(A/G, 12, 3)] = \$341,799$$

	P(0.5)	P(0.25)	P(0.25)	EV
No Change	310, 000	310, 000	310, 000	310, 000
Redesign	272, 453	290, 945	341, 799	294, 413

Therefore, we should go with the redesign.

7 Expected Value of Perfect Information

Suppose you know which future state will occur. Then you can compute the expected value given perfect information by multiplying the best outcome in each column of the payoff table by its probability and summing the products. The difference between the EV of the best project under risk and the expected value given perfect information is the value of perfect information.

Example:

$$EV\mid_{PI} = 0.5(272,453) + 0.25(290,945) + 0.25(310,000) = \$286,463$$

Thus the EV of perfect information is $EV\mid_{PI} - EV = \$286,463 - 294,413 = -\$7950$.

This is a negative cost that corresponds to savings of \$7950. You can spend up to this amount to determine which state will occur each time a decision is due.

8 Investment Risk Profiles

Example: Investment proposal with initial cost of \$200,000. Annual returns (R) are:

R	50,000	100,000	125,000
Probability	0.3	0.5	0.2

The possible durations (N) are:

N	2	3	4	5
Probability	0.2	0.2	0.5	0.1

There is no salvage value, the MARR is 12%, and R and N are independent. Provide an investment risk profile for this problem.

Outcome	$P(R \cap N)$	PW	$P(R \cap N) \cdot PW$
(50000,2)	0.06	-112045	-6723
(50000,3)	0.06	-73436	-4406
(50000,4)	0.15	-38014	-5702
(50000,5)	0.03	-5518	-166
(100000,2)	0.10	-24089	-2409
(100000,3)	0.10	53129	5313
(100000,4)	0.25	123972	30993
(100000,5)	0.05	188965	9448
(125000,2)	0.04	19889	796
(125000,3)	0.04	116411	4657
(125000,4)	0.10	204965	20497
(125000,5)	0.02	286206	5724
Total	1.00		\$58022

As, for example, $PW(P=0.06) = -200000 + 50000(P/A, 9, 2) = -\$112,045$.

The most likely outcome:

$$PW(P=0.25) = -200000 + 100000(P/A, 9, 4) = \$123,972.$$

Note: $E(PW) = \$58022$ is much less than the PW of the most likely outcome.

Diagram: Cumulative Probability - here the investment proposal has a probability > 0.4 of showing a loss.

Inv. Risk PW	Cumul. Prob.
-112045	0.06
-73436	0.12
-38014	0.27
-24089	0.37
-5518	0.40
19889	0.44
53129	0.54
116411	0.58
123972	0.83
188965	0.88
204965	0.98
286206	1.00

9 Other Decision Criteria

9.1 Expectation Variance (minimize variance)

(a) If $\sigma_A = \sigma_B$, choose the alternative with the larger mean.

(b) If $\sigma_A^2 < \sigma_B^2$ and $\mu_A \geq \mu_B$, then choose A

(c) If $\sigma_A^2 > \sigma_B^2$ and $\mu_A > \mu_B$, there is no easy choice.

Example:

	E(R)	σ
A	2320	1476
B	1620	2257
C	1550	1359

A will be preferred to B, but the choice between A and C is not clear.

9.2 Most Probable Future

The investment that has the greatest return for the most probable future is preferred.

Example:

PW	-1000	0	1000	2000	3000	4000
A	0	0.11	0.26	0.22	0.02	**0.39**
B	0.29	0.18	0.07	0	0	**0.46**
C	0.14	0.10	0.11	**0.37**	0.28	0

Here, the most probable future for A is P=0.39, for B is P=0.46, and for C is P=0.37. The PW of A in the most probable future is 4000, as is the PW of B, while the PW of C is 2000. We would then prefer B since it has the same PW as A, but with a higher likelihood.

9.3 Aspiration Level

Based on the minimum amount that will satisfy a decision maker.

Example: if the aspiration level is \$2000,

	Prob. of return ≥ 2000
A	$0.22 + 0.02 + 0.39 = 0.63$
B	0.46
C	$0.37 + 0.28 = 0.65$

Therefore, alternative C is preferred.

10 Variance of Composite Cash Flows

If $X = A + B$,

$$V(X) = V(A) + V(B) + 2Cov(A, B)$$

If A and B are independent, $Cov(A, B) = 0$ so

$$V(X) = V(A) + V(B)$$

For the product of random variables,

$$\sigma_{AB}^2 = \mu_A^2 \sigma_B^2 + \mu_B^2 \sigma_A^2 + \sigma_A^2 \sigma_B^2$$

and for a constant k,

$$V(kA) = k^2 \sigma_B^2$$

11 Risk of Aggregate Cash Flows

$$PW = X_0 + \frac{X_1}{(1+i)} + ... + \frac{X_N}{(1+i)^N}$$

$$EV(PW) = \mu_0 + \frac{\mu_1}{(1+i)} + ... + \frac{\mu_N}{(1+i)^N}$$

12 The Central Limit Theorem

Approximately:

$$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$

$$\begin{aligned} V(PW) &= \sigma_0^2 + \frac{\sigma_1^2}{(1+i)^2} + \frac{\sigma_2^2}{(1+i)^4} \dots + \frac{\sigma_N^2}{(1+i)^{2N}} \\ &= \sigma_0^2 + \sigma_1^2(P/F,i,2) + \sigma_2^2(P/F,i,4) + \dots + \sigma_N^2(P/F,i,2N) \end{aligned}$$

The probability that $PW < 0$ is given by

$$\begin{aligned} P\left(Z < \frac{0 - E(PW)}{\sqrt{V(PW)}}\right) &= P\left(Z < \frac{0-489}{258}\right) \\ &= P(Z < -1.69) = 0.46 \end{aligned}$$

if, for example, $E(PW) = 489$ and $V(PW) = (258)^2$

13 Investment to Avoid Risk

Modify exposure to risk in 2 ways:

1. alter the probabilities of the applicable future states (eg. irrigation project to eliminate danger of drought)

2. spread the risk (transfer the burden of risk), as when buying insurance to avoid disaster (pay premium for certainty)

14 Risk and Cash Flow Correlations

Firm considers two investments that would have an effect on its cash flow. Reducing the variance of the cash flow reduces the chance of bankruptcy. Cash flows are currently 200 in a "boom" economy (20% chance), 160 under normal conditions (60% chance), and 120 in a "bust" economy (20% chance). Investment 1 would increase cash flows by 20 in a boom, 40 in a bust and 30 otherwise. Investment 2 would increase cash flows by 40 in a boom, 20 in a bust and 30 otherwise.Evaluate the cash flow stability of the 2 proposals.

State of Economy	Prob.	Cash Flow 1	Cash Flow 2
Boom	0.2	20+200=220	40+200=240
Normal	0.6	30+160=190	30+160=190
Bust	0.2	40+120=160	20+120=140

$$\mu_1 = \mu_2 = 190$$

$$\sigma_1 = 18.97$$

$$\sigma_2 = 31.62$$

Proposal 1 reduces the range of variation from boom to bust from 200-120=80 to 220-160=60, while proposal 2 increase the range of variation to 240-140=100, so proposal 1 is preferred.